Toward a Coherent Mathematics Program

A Study Document for Educators

Response to the 2001
Elementary Mathematics Assessment

NOVA SCOTIA
Department of Education

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Elementary Mathematics Assessment

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Prepared by the Department of Education

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The department would also like to thank David McKillop, Mathematics Evaluation Consultant, Department of Education, for his leadership in mathematics education and his contribution to the preparation of this study document for Nova Scotia educators.

Contents

How to Use this Study Document

Teachers

1. While planning for a unit of work on a specific strand, read through the corresponding category sections of this document. Consider the comments and suggestions that are provided. Reflect on your own teaching practices and what you know about the students in your class. Consider the common misconceptions and difficulties that are presented. Incorporate the examples given for specific learning activities into your instructional plan.

2. Select some relevant questions from the 2001 Elementary Mathematics Assessment or similar questions and administer these questions in your class. Compare the results you obtain with the results reported provincially and/or check to see if your students have similar misconceptions or difficulties as those reported in this document. These questions may be conducted as a pre-test and then again as a post-test. Use the suggestions, comments, and examples of specific learning activities to remediate where necessary.

3. You are part of a continuum in the delivery of mathematics education at the elementary level. You should consider the information under the heading, Coherent Programming (pp. 3–5). Reflect on your contribution to the delivery of a coherent mathematics program to students at your school and set professional goals in this regard.

School Staffs

1. All the mathematics teachers within a school should work toward the goal of providing students with a coherent mathematics program. To this end, a whole school approach is required. Arrangements should be made to meet and consider the comments, suggestions, and specific learning activities in this document.

2. The initial meeting should have the expressed purpose of addressing the six recommendations that follow the question on page 3; “What can be done to provide students with a coherent program?”

3. Subsequent meetings should address the collective role of mathematics teachers in relation to the comments, suggestions, and specific learning activities that are presented under each of the category headings in this document.

4. The information under each category in this document may also provide a focus for professional development for the school staff. It could also be a source to target specific curriculum needs for the staff.

School Boards

1. Board curriculum supervisors/consultants and mathematics committees/leadership teams should consider how they can provide help and support to individual teachers, groups of teachers, and school staffs as they work toward the major goal of providing students with a coherent mathematics program.

2. Comments, suggestions, and specific learning activities found in the category sections of this document are an excellent source of material for workshops or discussions with individual or groups of teachers including school staffs.

Introduction

This document was prepared by the Department of Education for board administrators, principals, and teachers. It contains an analysis of the results of the 2001 Elementary Mathematics Assessment. It identifies weaknesses in the mathematics education of our elementary students and provides classroom teachers with suggestions to plan and deliver relevant and effective instruction to improve their students' learning and achievement in mathematics.

It is especially important to note that while the assessment was administered in grade 5, it included questions on materials covered at other grades in the elementary mathematics program. Thus, the achievement of grade 5 students in this assessment is a shared responsibility among the mathematics teachers in a school. All mathematics teachers, under the leadership of the school principal, should give collective consideration to the information contained in this document. In this way each elementary school will ensure that mathematics is delivered as a coherent program.

This document reports on the assessment in six categories representing clusters of questions related to specific curriculum outcomes under the general curriculum outcomes of the Atlantic Canada mathematics curriculum. For each question, readers are provided with the target grade, the specific outcome, a brief description of the concept/skill involved, and the results for the students in Nova Scotia. In each of the six categories, there are general comments on the results, references made to previous suggestions in the *Report on Field Tests: What We Learned* document, and additional suggestions for teaching and learning.

Readers are advised to consider the *Elementary Mathematics Program Assessment 2001 Results for Nova Scotia*, which is a more general report prepared for board members, parents, advisory groups, and the public at large. The general report gives an overview of the results. As well, the companion document, *Report on Field Tests: What We Learned* (December 2000), should be referenced for the comments and suggestions made in it.

The comments and suggestions in this document complement the information provided with the specific curriculum outcomes in the Atlantic Canada mathematics curriculum under the headings *Elaboration-Instructional Strategies/Suggestions* and *Worthwhile Tasks for Instruction and/or Assessment*. Thus, the information in this document may be used at all levels: board, school, and classroom to improve mathematics education for students in Nova Scotia. It is recommended that individual teachers or groups of educators engage with the contents of this document to isolate concerns, identify needs, and develop strategies for the continuous improvement in the teaching and learning of mathematics.

The assessment will be conducted again in 2003.

Coherent Programming

Students need a coherent program in order to fully benefit from the Atlantic Canada mathematics curriculum. Coherence is required in both what is taught and in the way in which it is taught. The Atlantic Canada mathematics curriculum documents outline specifically what is to be taught at each grade level and give suggestions for how to teach the various prescribed concepts and procedures. The mathematics curriculum was carefully constructed to provide a logical sequence of development of mathematical ideas from grade to grade successively to the end of the public school program.

What can be done to provide students with a coherent program?

1. Teachers at all grade levels need to plan and organize their mathematics program based on the outcomes assigned to their specific grade(s), judiciously selecting text and other resources to match these outcomes.

   Vigilance is required to keep the mathematical activities focussed on the prescribed learning outcomes and focussed on what students need to be doing in order to grow in their understanding and abilities with regard to these outcomes.

2. Teachers from grades primary to 6 within a school should work co-operatively to plan the mathematics program. Typically, each teacher is responsible for the delivery of one-seventh of the elementary program. Each is part of a continuum of mathematics instruction. Thus, each teacher's role is a critical link in the process of ensuring a strong elementary program. If there is more than one teacher at a grade level within the school, then these teachers should also plan and organize together, sharing ideas and activities for the betterment of their students.

   If there are teachers in schools with expertise in mathematics education, then these teachers should be viewed as valuable resources, and their knowledge and understanding should be utilized.

3. Teachers should not hesitate to acknowledge their own deficiencies in mathematics. Rather, they should be keen to develop their own personal understandings of the mathematics topics they are expected to teach. Since teaching for understanding is a major goal of the Atlantic Canada mathematics curriculum, it is important that teachers have that understanding themselves in order to structure lessons to help students gain clarity of these same topics.

4. Teachers at all grade levels need to understand, and endorse in their practices, what it means to *do* mathematics. Students, for their part, must recognize that mathematics is about thinking and logical reasoning, looking for patterns and relationships, solving problems, and communicating and sharing ideas and strategies. Such recognition can be achieved only if teachers engage their students in these activities on a regular basis. On the other hand, if the students' classroom experience is one of listening to ideas being explained by the teacher, copying set procedures, and rehearsing a *rule* by applying it again and again to similar questions, then they will acquire a view of what it means to *do* mathematics that is very different from what is articulated by the Atlantic Canada mathematics curriculum.

5. Teachers at all grade levels need to understand, and endorse in their practices, what it means to teach concepts, procedures, and strategies in mathematics, so that students get a consistent message in their day-to-day, week-to-week, month-to-month, and year-to-year activities in mathematics. Simply stated, students have a thorough understanding of a concept if they recognize exemplars and non-exemplars of that concept in any of its representations (contexts, symbols, concrete models, pictorial models); if they know and use the language associated with that concept; if they can go from one representation of the concept to another; if they make connections to other concepts; and if they can apply the concept in new and novel situations.

   Representations: Solving story problems (contexts) with concrete models and pictures is a crucial first step in the teaching of a concept with the associated symbols introduced as a form of communication of the idea embedded in these story problems and concrete/pictorial models. With each of these representations, students need to be actively engaged—thinking and communicating—to perceive the patterns and relationships that are the essence of the concept.

   This approach to the teaching and learning of mathematics is in stark contrast to older methodologies that promoted the mastery of the procedural (symbolic) aspects of the concept before students applied these procedures to a limited number of associated story problems. A quick perusal of texts that were used to support older mathematics programs reveals that mastery of procedures rather than solving problems was the central focus of the mathematical activity. Teachers must use resources that support the current program.

   Procedural Strategies: In the Atlantic Canada curriculum, students are expected to learn and apply a variety of mental math, estimation, and pencil-and-paper strategies with a focus on understanding these strategies. Such understanding is developed through connections to concrete and pictorial models, through extending previously learned procedures, and through discussions of student-invented, non-standard, and standard algorithms.

Students must focus on a specific strategy long enough to grasp its inherent logic and to gain facility with it. Each strategy must be practised in isolation and then combined with other strategies before it can become part of the students' repertoire of strategies. Again, this is in stark contrast to some older teaching methodologies where the emphasis was on paper-and-pencil methods, which for any given operation only one strategy was shown and explained. More often than not, more emphasis was being placed on memorizing the steps of the procedure than on understanding the reasoning behind these steps.

6. In order to accurately gauge the progress of students, classroom assessment practices must reflect the curriculum foci of understanding concepts and procedures, and of solving problems. Whether the assessment is through portfolios, projects, observations, interviews, or pencil-and-paper instruments, the questions asked should provide responses that give teachers insight on an individual student's growth and development of understanding and problem-solving ability.

   The assessment of specific outcomes should not be restricted to the occasion when the outcomes are being studied. Periodically, teachers should examine students' long-term retention of major concepts and procedures through review exercises and/or assessments. As well, every opportunity should be taken to integrate previously learned concepts with concepts currently being studied. The goal is to provide students with a coherent program and to prepare them for the next grade where they will continue to develop their understandings and abilities.

Teachers have the principal role in providing students with a coherent mathematics program. Implementation of these six recommendations will require the support of all teachers assigned to teach mathematics. In turn, teachers need the understanding and support of the school boards and administrators in their quest to improve the teaching and learning of mathematics in accordance with the Atlantic Canada mathematics curriculum.

Teachers must have adequate and appropriate resources, both print and concrete, to carry out the program. They need ongoing professional development opportunities to increase their personal understandings of mathematics and to broaden their teaching methodologies. All stakeholders must explore a variety of ways to meet these professional development demands.

Category 1
Number Concepts and Patterns—GCO A and GCO C

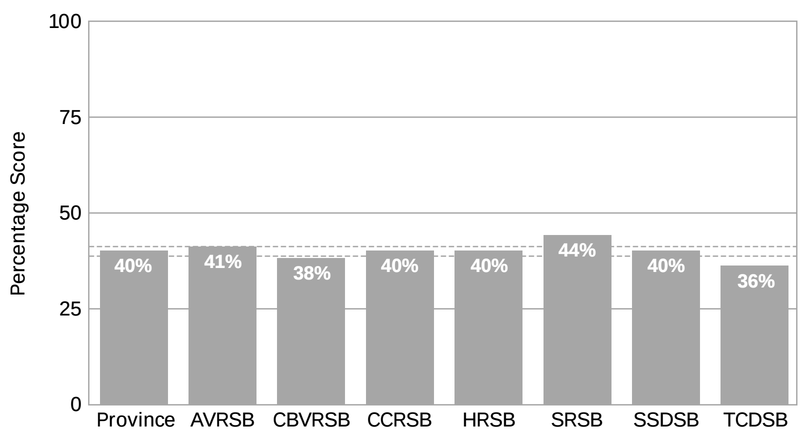
The questions in the category for Number Concepts and Patterns are based on General Curriculum Outcomes (GCO). The questions accounted for 16 points out of the 125 total points on the assessment. The distribution of the questions across the assessment and the results are shown in the table on the next page. The table also designates the Specific Curriculum Outcome (SCO) for each question.

The questions in this category assessed students' knowledge and comprehension of place value, whole numbers, common fractions, decimals, and number patterns. To assess comprehension of fractions and decimals, emphasis was given to concrete and pictorial representations.

The graph below shows the provincial mean percentage score for number concepts and patterns as 39.8 percent and the school board mean percentage scores ranging from a low of 35.8 percent to a high of 43.6 percent. The parallel broken lines on the graph provide the confidence interval for the provincial results. The confidence interval represents the high- and low-end points between which the actual results fall, 95 percent of the time.

AVRSB—Annapolis Valley Regional School Board
CBVRSB—Cape Breton-Victoria Regional School Board
CCRSB—Chignecto-Central Regional School Board
HRSB—Halifax Regional School Board
SRSB—Strait Regional School Board
SSDSB—South Shore District School Board
TCDSB—Tri-County District School Board

Number Concepts and Patterns



Percentage scores rounded

Table 1: Number Concepts and Patterns

Part	Question Number	Point Value	Target SCO	Description of Skill/Knowledge/Concept	Mark: % Achieving*
2	1	1	5A1	Recognize a numeral for an expanded form	1: 60.8%
	2	1	4A2	Recognize a decimal for a base–10 picture	1: 75.9%
	3	1	5A7	Recognize ways to read a decimal	1: 55.6%
	4	1	5A9	Recognize the largest of four decimals	1: 20.1%
	5	1	4A1	Recognize the location of a fraction on a number line	1: 41.6%
	6	1	4A1	Recognize a fraction to describe a picture	1: 47.9%
	7	1	5A3	Recognize pictures of equivalent fractions	1: 32.7%
	40	1	5C5	Recognize an extension of a pattern for division	1: 34.5%
5	1	1	4A1	Draw a rectangle; show two-thirds shaded	1: 28.2% 0.5: 20.2%
	5	1	5A10	Choose a denominator to satisfy a condition	1: 17.8% 0.5: 3.5%
	6	1	4A1	Name a number for a pattern block display	17.8%
	7	1	4A2	Name decimals for base–10 block displays	1: 21.4% 0.5: 7.0%
	8	1	5A2	Name a decimal for an indicated point on a number line	1: 7.7% 0.5: 1.7%
6	5	3	5C3	Solve a problem by detecting a pattern and extending it	3: 34.9% 2.5: 13.4% 2: 4.2% 0: 33.1%

*These percentages are based on the provincial sample of grade 5 students.

Comments

The results of Part 6: Question 5 were encouraging. This question was a non-routine problem for most grade 5 students. Nevertheless, students performed better on this question than on most of the other more routine constructed-response questions.

The overall results for GCO A indicates that substantially more conceptual development of both common fractions and decimals is required. It was observed that

- students had the most difficulty generating the correct symbols for concrete displays of these types of numbers (Part 5: Questions 6 and 7), but they were better able to recognize appropriate symbols for pictorial displays (Part 2: Questions 2 and 6; Part 5: Question 1)

- students were better able to recognize the correct location on a number line for a common fraction (Part 2: Question 4) than they were able to generate the correct decimal for an indicated point on a number line (Part 5: Question 8)

Perhaps the most telling indicator of poor conceptual understanding of decimals is the result in choosing the largest of four decimals—0.2, 0.22, 0.022, and 0.202—(Part 2: Question 4). Forty-three percent of the students chose 0.202, which suggests that they were treating the decimals as whole numbers, having no regard for the place value. Another 23 percent chose 0.2, which suggests an over-generalization that "10ths are larger than 100ths and 1000ths" or that visually the symbol is closer to the ones place. Regardless of the misconception, it is clear that

- students were not visualizing the relative number of base-10 blocks that would represent each number or the relative position of each number on a number line—both good conceptual strategies that would be the result of teaching decimals well

It is also interesting to note that more students were able to associate pictures of common fractions and their symbols (Part 2: Question 6 and Part 5: Question 1) than could associate concrete models and their symbols (Part 5: Question 6). Likely it is because students could clearly see the association of the parts and the whole in the pictures, while they had to establish the relationship between the two blue pattern blocks and the yellow block in the concrete model. As well, the concrete model involved mixed numbers, while the visual models did not. However, this result speaks to the necessity of having both concrete and pictorial models in the development of fractions—one is not a substitute for the other.

It was observed that more students were able to connect pictorial area models and common fractions than were able to connect area models of equivalent fractions (Part 2: Question 7). Such area models should be at the core of teaching equivalent fractions for understanding.

Previous Suggestions

The suggestions presented in *Report on Field Tests: What We Learned* distributed to teachers in 2000 should be reviewed as they continue to be valid in relation to the results of the 2001 Elementary Mathematics Assessment. Teachers are encouraged to review pp. 6–9 in that document for comments and suggestions related to (1) using number lines to represent decimals, (2) using base-10 blocks to represent decimals, (3) using the convention for writing numbers, (4) comparing common fractions using conceptual methods, and (5) using mathematics vocabulary.

Additional Suggestions

The following suggestions, based on the results of the 2001 Elementary Mathematics Assessment, are provided for teachers to assist their students to improve their mathematical skills and understanding.

1. Grade 3 teachers must carefully plan their students' first introduction to the concept of decimal 10ths. The *Elaboration* section of SCO, A7, provides many suggestions for the use of contexts to create the need for students to work with decimals—such contexts are a crucial part of the development of decimal concept in the minds of students. The move from whole numbers to decimals is a critical point in the development of the base-10 system. Decimals require that students internalize the relationship between the parts they observe or count and that which is the whole.

   - For example, if five small cubes from the base-10 blocks are placed in front of students and they are asked, What do you see?, they would count them and say, Five cubes. However, if they are asked, What part of this rod do you see?, they count the cubes, get five, but have to consider how these five cubes relate to the rod as a whole before they say, 5-10ths.

   Students need many experiences making these part-whole relationships. Teachers should periodically show students a few objects and ask them, How many things do you see?, and then ask them, When might these objects be 10ths?

   - For example, four pieces of paper could be used. If students were counting pieces of paper, the answer is four, but, at the same time, this quantity of paper could be 4-10ths of a pad of 10 sheets of paper. Grade 4 and 5 teachers could extend this idea to help students appreciate that the same four sheets of paper could be 4-100ths of a pad of 100 sheets and 4-1000ths of a package of 1000 sheets.

   In short, counting objects is one thing, putting them in a fractional relationship is another thing altogether.

2. Teachers must prepare students to count decimals in a variety of ways, such as 0.1, 0.2, 0.3, ..., and 0.3, 0.6, 0.9, ... In this way students will experience what happens when they get more than 9-10ths. (Note: the teacher must say 10ths, such as 1-10th, 2-10ths, 3-10ths,...). As well, students must be prepared to name the quantities greater than 9-10ths in two different ways.

    - For example, in the first count described above, they would say, 10-10ths and 1; and for the second skip-count, they would say, 12-10ths and 1 and 2-10ths.

    Students must be provided with meaningful classroom activities that combine context, concrete, symbol, and verbal representations for 10ths. One such activity is described below.

    Provide students with calculators. Describe a situation where 10 chocolates are in a package and place a rod from the base-10 blocks on the overhead to represent this package of chocolates. Show students 2 small cubes to represent single chocolates and ask, What part of a package do these 2 chocolates represent? Instruct them to turn on their calculators, press the plus sign button, enter 0.2, and press the equal sign button. As you place 2 more cubes on the overhead, have the students press the equal sign button again, and say, 4-10ths of a package. Continue adding small cubes, 2 at a time, with students pressing the equal sign button, and chanting, 6-10ths of a package, and then, 8-10ths of a package. Before you add 2 more cubes and they press their equal sign buttons, ask them to describe what they think will come next. (Don't be surprised if students will expect to see 0.10 on the calculator.) Add the 2 cubes, have them press the equal sign button, and discuss the 1 that appears on the screens. Replace the 10 small cubes on the overhead with one rod. Continue the activity, adding the small cubes, 2 at a time, pressing the equal sign buttons, and chanting, 1 and 2-10ths packages, 1 and 4-10ths packages, ..., 1 and 10-10ths or 2 packages, ... and so on.

    Similar counting activities can be performed with other contexts, other concrete material, and other amounts. It can also be done as a partner activity where one student has the calculator, the other student has the base-10 blocks, and they count together. In grades 4 and 5, similar activities can be done counting 100ths and 1000ths.

3. Compatibles should be extended to work with 10ths by finding the pairs of 10ths that make 1.

    - For example, 0.4 and 0.6 are compatible pairs because they make 1. In grade 3, students should explore and learn to recognize all such compatible pairs. Similarly, in grades 4 and 5, teachers can help students explore and learn to recognize pairs of 100ths that make 1 and pairs of 1000ths that make 1.

These abilities transfer to a make-1 strategy in addition.

- For example, if teachers ask students to add 0.7 and 0.5, the students might think of it as (0.7 + 0.3) + 0.2.

4. Teachers must emphasize with their students the difference between comparing 32 with 291 and 0.32 with 0.291. It has been observed that when students are asked to compare two or more decimal numbers, they continue to use a generalized strategy from their work with whole numbers—the number that uses the most digits is the greater. This misconception must be challenged as early as grade 3 by asking students to compare numbers such as 21 and 20.7, and then continuing in grade 4 with comparisons between 0.2 and 0.19 and in grades 5 and 6 with comparisons between 0.32 and 0.291. If students were encouraged to do such comparisons by visualizing the base-10 block representations of the numbers and/or their relative positions on a number line, many comparison errors would be minimized. At every opportunity, teachers should remind students that symbols represent something and that "something" should be visualized and worked with rather than dealing with the symbols in isolation.

5. Teaching common fractions requires the same treatment as that recommended for decimals. A common fraction represents the relationship between parts and the whole. Again, students count the parts and then have to establish the relationship between these parts and the whole. Because the whole is more apparent in area models such as fraction pieces and pattern blocks, it is recommended that students' first experiences be with those models. Following this, students should move on to set models such as two-colour counters and straight-line models such as straws and number lines.

   An added difficulty with common fractions is associated with the notation used to represent the relationship between parts and the whole. Students' experiences with whole numbers causes them to view the numerator and denominator of a fraction as two whole numbers, independent of one another, rather than as a relationship. They need many experiences and reminders to establish in their minds that the denominator is the fraction's *family* name. When the students initially record fractions, it is suggested that they write what they say—for example, 3-fourths, 2-thirds, and 5-eighths—before they use the numerator and denominator symbolization.

6. Teachers must provide students extensive opportunities for counting experiences with common fractions. One such activity is described below.

   Tell the students that the yellow pattern block represents 1. Then ask them what the blue block would represent. Once you have established that the blue block is 1-third, ask students to count 8 blue blocks as you place them one at a time on the overhead projector. You should hear, 1-third, 2-thirds, 3-thirds, 4-thirds, 5-thirds, 6-thirds, 7-thirds, 8-thirds. Stop at this point and establish that 3-thirds make a hexagon (yellow block); therefore, 3-thirds make 1 and 8-thirds would make 2 and 2-thirds. Continue placing the blue blocks one at a time on the overhead projector with students continuing to count whole numbers and thirds. (2 and 3-thirds or 3, 3 and 1-third, 3 and 2-thirds, ...).

   This counting can include recording the counts as fractions and as mixed numbers.

   $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}, \frac{6}{3}, \frac{7}{3}, \frac{8}{3}, \frac{9}{3}, \frac{10}{3}, \frac{11}{3}, \frac{12}{3}, \ldots$

   $\frac{1}{3}, \frac{2}{3}, 1, 1\frac{1}{3}, 1\frac{2}{3}, 2, 2\frac{1}{3}, 2\frac{2}{3}, 3, 3\frac{1}{3}, 3\frac{2}{3}, 4, \ldots$

   The activity can be repeated using other pattern blocks to count different fractions, using a different block to represent 1, and using other models.

   Fraction-counting activities help reinforce that fractions are relationships, that the numerator is what is counted and changing, and that the denominator remains unchanged. As well, these counting activities are a good foundation for mixed numbers.

7. Students should have experiences working with the symbols for fractions to satisfy different conditions, such as the ones described below.

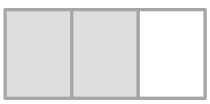
   $\frac{7}{\square} \quad \frac{7}{\square} \quad \frac{7}{\square}$

   Given the three fractions above, choose a denominator for the first fraction so the fraction is close to 1, a denominator for the second so the fraction is close to 0, and a denominator for the third so the fraction is close to one-half.

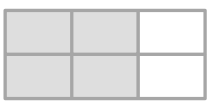
$\frac{\square}{25}$ $\frac{\square}{25}$ $\frac{\square}{25}$

Given the three fractions above, choose a numerator for the first fraction so the fraction is a little less than one-half, a numerator for the second so the fraction is close to 0, and a numerator for the third so the fraction is close to one-fourth.

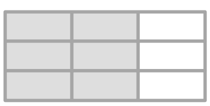
8. It might appear that teaching equivalent fractions is rather simple. After all, learning "to multiply top and bottom by the same number" is not a difficult procedure. However, teachers want students to understand what this multiplication is actually doing and that while the size of the numerator and denominator are changing, the relationship between them is not. A visual representation of what is happening is illustrated below.



$\frac{2}{3}$



$\frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$



$\frac{2}{3} \times \frac{3}{3} = \frac{6}{9}$

In the second rectangle, the whole rectangle is partitioned into twice as many pieces as in the first rectangle, so there are twice the number of pieces shaded, but the amount of area shaded is the same.

In the third rectangle, the whole rectangle is partitioned into three times the number of pieces, so there are three times the number of pieces shaded, but the amount of area shaded is the same.

In short, the effect of multiplying the numerator and denominator of a fraction by the same number is to make more pieces out of the same whole so that each piece becomes smaller. Therefore, more of these smaller pieces will be needed to cover an equivalent area.

It is also clear that multiplying the numerator and denominator of a fraction by the same number is equivalent to multiplying by 1, because the whole does not change. (Note: students know that multiplying a whole number by 1 does not change the look of its symbol or its picture representation, so they must be convinced by these fraction pictures that this process is in fact multiplying by 1, even though the symbols look different.)

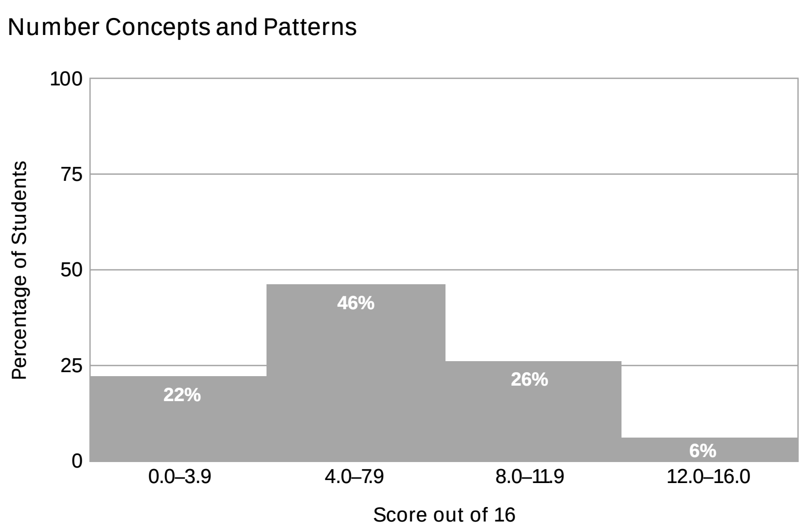
Students should draw picture representations when they are finding equivalent fractions until they are competent in visualizing and explaining the procedure. Their understanding of equivalent fractions is critical to the further development of other ideas about fractions and to the elimination of other fraction misconceptions.

Additionally, there are other models that teachers can use to illustrate equivalent fractions. These models include fraction pieces, pattern blocks, and simple paper folding.

- For example, a teacher may ask students to fold a sheet of paper in half twice, unfold it to show fourths, colour one-fourth, and re-fold the paper. If students are then asked to fold the paper in half again and unfold it, they will have eighths and will see that the one-fourth they coloured now shows two-eighths.

Provincial Sample Quartile Results for Category 1

The histogram below shows the percentage of students in the provincial sample that scored in each quartile of the 16 points accounted for by the assessment questions for GCO A and C, Number Concepts and Patterns.



Category 2
Computational Tasks—GCO B

The questions in this category accounted for 20 points out of the 125 total points on the assessment. The distribution of the questions across the assessment and the results are shown in the table on the next page.

The questions on computational tasks assessed students' knowledge of numbers facts, mental math strategies, estimation strategies, and paper-and-pencil procedures. For the most part, the questions required students to find or recognize correct numerical answers to computations. This category and Category 3 both report on GCO B of the Atlantic Canada mathematics curriculum.

The graph below shows the provincial mean percentage score for this category of the assessment as 50.6 percent and the school board mean percentage scores ranging from a low of 46.9 percent to a high of 53.2 percent.

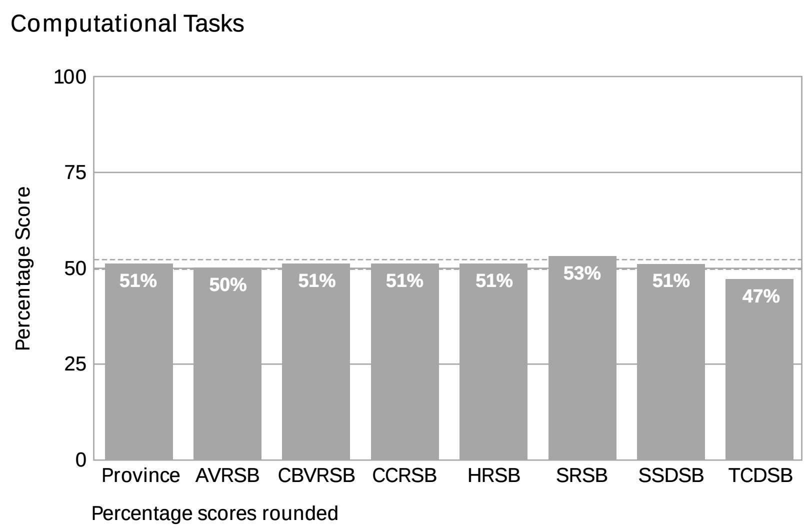


Table 2: Computational Tasks

Part	Question Number	Point Value	Target SCO	Description of Skill/Knowledge/Concept	Mark: % Achieving*
1	1–11 12–18	6	various in 3B, 4B, 5B	1–11: Know numbers facts for + / - / x / ÷ Getting 10 or 11 out of 11: Getting 8 or 9 out of 11: 12–18: Apply mental math strategies Getting 6 or 7 out of 7: Getting 4 or 5 out of 7:	 1: 38.7% 1: 25.3% 1: 15.5% 1: 30.6%
2	31	1	3B6	Recognize missing digits in subtraction	1: 40.2%
	33	1	5B2	Recognize an alternative multiplication procedure	1: 39.2%
	38	1	4B3	Recognize notations for division	1: 58.9%
3	1(A)	1	4B1	Paper-and-pencil question: 5238 + 12 993	1: 73.6%
	1(B)	1	5B1	Paper-and-pencil question: 2.356 + 0.89	1: 51.6%
	2(A)	1	4B1	Paper-and-pencil question: 8337 – 949	1: 61.3%
	2(B)	1	5B1	Paper-and-pencil question: 5.85 – 1.392	1: 24.8%
	3(A)	1	5B2	Paper-and-pencil question: 5638 x 4	1: 60.8%
	3(B)	1	5B3	Paper-and-pencil question: 15 x 17	1: 47.2%
		1	5B3	Relate rectangular array to 3(B)	1: 8.9%
	4	1	4B5	Paper-and-pencil question:	1: 43.2%
4	41	0.5	4B14	Recognize an estimate of 488 x 6	1: 31.6%
	42	0.5	5B11	Recognize an estimate of 48 x 41	1: 31.9%
	43	0.5	4B14	Recognize an estimate of 652 ÷ 3	1: 52.7%
	44	0.5	4B15	Recognize an incorrect sum/difference	1: 45.7%
	45	0.5	4B13	Recognize an estimate of 77 + 62 + 25 + 38	1: 37.9%
	46	0.5	5B5	Recognize an estimate of 4–10ths of $79	1: 54.3%

*These percentages are based on the provincial sample of grade 5 students.

Comments

Overall the results for the pencil-and-paper procedures were disappointing, but it is of interest to note that students did better on the multiplication and division questions in the 2001 assessment as compared to their performance on similar questions on the field tests in 2000. Similarly, there was improvement noted in the results for the estimation questions.

For the pencil-and-paper questions, markers at the central scoring session accepted both standard and non-standard algorithms. However, it was observed that most students used or attempted standard procedures only. It was also noted that the types of errors observed were the same as those commented on following the field tests, and reported in 2000 in *Report on Field Tests: What We Learned* (pp. 9–12). The discussion in the field test report continues to be current and teachers should review the document as part of their instructional planning process.

Previous Suggestions

There are seven comments and suggestions in the 2000 field test report, *Report on Field Tests: What We Learned* (pp. 9–12). They deal with

- fact learning
- alignment of decimals in pencil-and-paper addition and subtraction computations
- estimation of products
- mental math estimation
- decimal products and quotients
- symbolic representations of division
- other common errors in subtraction

Additional Suggestions

The following suggestions, based on the results of the 2001 Elementary Mathematics Assessment, are provided for teachers to assist their students to improve their mathematical skills and understanding.

1. The development of pencil-and-paper procedures must include contexts with models using conrete materials and pictures. This must be done to bring clarity to the concept that the symbols are communicating and to the place values of the digits being manipulated. (Note: see more suggestions in the comments for Category 3.)

2. Teachers should ensure that students develop estimation strategies for a particular procedure before the development of the actual algorithm. As well, students should make an estimation before any calculation is done on a calculator or by a pencil-and-paper procedure. Teachers must accept the fact that estimation strategies are taught strategies—they are not a by-product of pencil-and-paper procedures. Likewise, concrete materials and pictures serve to illustrate the logic of these strategies so that students understand why and how the strategies work. Each strategy should be practised first in isolation and then with other accumulated strategies. Teachers should engage their students in a discussion of the strategies that are being used as they participate in estimation exercises.

3. Teachers should also encourage their students to invent mental strategies and to share them with their classmates. Discussions among students are critical for understanding these strategies. When there are two or more strategies that could

be used, each strategy should be discussed and practised; students could then be encouraged to use the one that makes the most sense to them. As stated before, concrete materials and pictures serve to illustrate the logic of these strategies.

It is through the development of estimation and mental math strategies that students develop the type of flexibility and understanding that are at the core of operation sense.

4. Non-standard pencil-and-paper algorithms have an important role in the development of procedures, whether or not students choose to use them. For example, consider the sum of 478 and 364, which is found by three different non-standard algorithms in the illustration shown.

Questions: 478 + 364

$$\begin{array}{r} 478 \\ +\,364 \\ \hline 700 \\ 130 \\ 12 \\ \hline 842 \end{array} \qquad \begin{array}{c} 400 + 70 + 8 \\ +\,300 + 60 + 4 \\ \hline \cancel{700} + \cancel{130} + \cancel{12} \\ 800 + 40 + 2 \\ 842 \end{array} \qquad \begin{array}{r} 478 \\ +\,364 \\ \hline \cancel{7}\cancel{3}2 \\ 842 \end{array}$$

- In the first procedure, the student started with the 100s, then the 10s, then the 1s, and added these three together to get 842.

- In the second procedure, the student wrote each number in expanded form, added the 100s, 10s, and 1s, regrouped each to get the expanded form of the answer, and wrote the answer in standard form.

- In the third procedure, the student started with the 100s to get 7, then added the 10s to get 13, which resulted in the 7 becoming 8 because of the additional 100, and finally added the ones to get 12, which resulted in the 3 becoming 4 because of the additional 10.

It is interesting to note that each of these three non-standard algorithms requires a better understanding of place value than does the standard algorithm. All three of these procedures are a result of a direct recording of finding the sum with base-10 block representations of the numbers if the flats had been combined first, then the rods, and finally the small cubes.

Two non-standard multiplication algorithms to find the product 16 x 23 are shown below.

$$\begin{array}{r} 23 \\ \underline{\times\, 16} \\ 18 \\ 120 \\ 30 \\ \underline{200} \\ 368 \end{array} \qquad\qquad \begin{array}{r} 23 \\ \underline{\times\, 16} \\ 200 \\ 30 \\ 120 \\ \underline{18} \\ 368 \end{array}$$

Both of these procedures show the four sub-products.

- The first procedure starts with the product of the 1s and concludes with the product of the 10s.

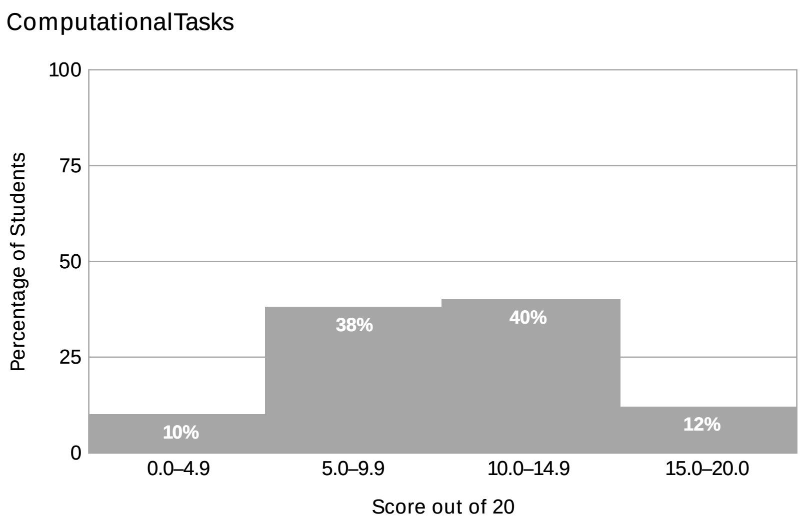
- The second procedure starts with the product of the 10s and concludes with the product of the 1s.

Neither procedure requires the student to do any adding until the final step—a fact that will reduce a number of the errors that occur in the standard algorithm. Teachers must ensure that students start with one of these two algorithms. The algorithms clearly illustrate the connection to the array representation that is the ideal model for the development of the logic of the procedure (Note: see the suggestions in Category 3: Operation Concepts and Applications).

When students are comfortable with one of these non-standard procedures, they should be introduced to the standard algorithm. Nevertheless, students will continue to benefit as they use non-standard procedures.

Provincial Sample Quartile Results for Category 2

The histogram below shows the percentage of students in the provincial sample that scored in each quartile of the 20 points accounted for by the questions dealing with procedural tasks.



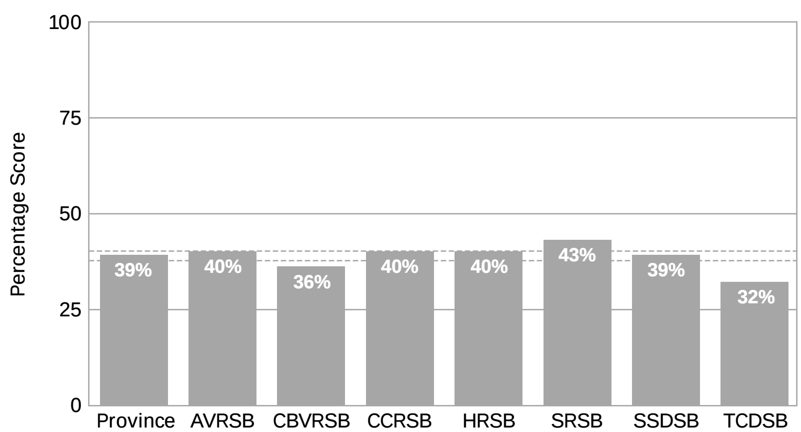
Category 3
Operation Concepts and Applications—GCO B

The questions in the category Operation Concepts and Applications are based on GCO B. The questions accounted for 26 points out of the 125 total points on the assessment. The distribution of the questions across the assessment and their results are shown in the table on the next page.

The questions in Category 3 assessed students' comprehension of procedures and their abilities to apply the operations of addition, subtraction, multiplication, and division to routine and non-routine problems. The routine problems involving one- or two-step solutions are typically associated with a specific concept/operation. Students should have encountered many of these routine types of questions during the study of each concept/operation.

The graph below shows the provincial mean percentage score for this category as 38.8 percent and the school board mean percentage scores ranging from a low of 32.3 percent to a high of 42.8 percent.

Operation Concepts and Applications



Percentage scores rounded

Table 3: Operational Concepts and Applications

Part	Question Number	Point Value	Target SCO	Description of Skill/Knowledge/Concept	Mark: % Achieving*
2	32	1	3B4	Recognize an addition sentence for a problem	1: 35.0%
	34	1	5B9	Recognize a Base–10 display for a multiplication problem	1: 34.1%
	35	1	5B9	Recognize the answer to a multiplication problem	1: 69.2%
	36	1	5B9	Recognize the answer to a two-step multiplication problem	1: 31.0%
	37	1	5B9	Recognize the answer to a multiplication (decimal) problem	1: 30.5%
	39	1	4B10	Recognize the answer to a division problem	1: 49.1%
4	47	0.5	3B9	Estimate the answer to a subtraction problem	1: 44.1%
	48	0.5	4B13	Estimate the answer to an addition problem	1: 54.8%
	49	0.5	3B9	Estimate the answer to an addition problem	1: 45.3%
	50	0.5	4B13	Estimate the sum of a Base–10 block display	1: 30.7%
6	1	3	4B11	Solve a two-step problem	3: 28.7% 2.5: 8.9% 2: 1.9% 0: 30.9%
	3	3	4B1	Spot and explain the error in addition of two decimal 10ths	3: 9.5% 2.5: 9.4% 2: 13.2% 0: 44.4%
7	7	3	4B1	Explain the base-10 block subtraction of two decimals to 100ths	3: 4.6% 2.5: 8.4% 2: 4.4% 0: 26.5%
	8	3	3B4/5 4B11 5B8/9	Create a story problem that satisfies two conditions	3: 26.9% 2.5: 4.3% 2: 18.1% 0: 36.7%
8	11	6	5B9 5D5	Solve a problem involving costs for hardwood floors in two rooms	6: 19.6% 5: 9.8% 4: 13% 0: 48.6%

*These percentages are based on the provincial sample of grade 5 students.

Comments

Using mathematics to solve problems in everyday life is a central focus of the Atlantic Canada mathematics curriculum. The application of mathematics to real-life situations gives students a reason for studying a concept. It is also critical for their comprehension of that concept. Each specific curriculum outcome that addresses problem solving includes the expectation that students will also create problems. The results of the 2001 assessment indicate that a substantial amount of work is needed to bring the application of mathematics to the forefront of classroom activities. The role of each mathematics teacher in Nova Scotia elementary schools is critical to improve the mathematics education of students in this regard. A whole school effort is required.

While only about 40 percent of the students scored two to three out of three on Part 6: Question 1, the variety of strategies used was encouraging. Question 1:

Mr Gordon is renting buses to take 148 students to the provincial science fair. Each bus has 43 seats. Mr Gordon plans to rent 4 buses, so there will be extra seats for parents and teachers. How many extra seats will there be?

It was observed that

- some students multiplied 43 by 4 to get the total number of seats and subtracted 148
- other students divided 148 by 4 to get the number of students for each bus, subtracted the 37 from 43 to get the number of extra seats in each bus, and finally multiplied 6 by 4 to get the total
- Still other students, although a minority, drew the 4 buses with the 43 seats and distributed the 148 students to see the number of extra seats

In Part 7: Question 8 students were asked to create a story problem that would involve the numbers 4, 12, and 20, and that would require someone to do at least two different operations to find its solution. Question 8 was successfully answered by 31.2 percent of students using skills and concepts from grades 2 to 5. It is disappointing, however, to report that 37.8 percent of the students who took the assessment did not attempt the question or did not appear to know what constituted a story problem—a context, given information, and a question. The rest of the students made errors in meeting the two required conditions.

In Part 8: Question 11 students were required to apply the concept of area of a rectangle as well as operation skills. This combination of ideas made for a higher-level–thinking question, albeit of a very practical kind. However, this integration of concepts probably accounts for the 48.6 percent of students who made either a false attempt or no attempt to answer the question.

The Atlantic Canada mathematics curriculum sets expectations that students not only learn but also understand procedures. Part 6: Question 3, and Part 7: Question 7 were two questions designed to probe students' understanding of addition and subtraction of decimals.

- Question 3 asked students whether the shown addition of two decimal 10ths was correct, to explain how they knew, and to draw a picture to explain the addition. A surprising result was that over 40 percent of the provincial sample said the incorrect answer was correct—this in spite of having calculators to check their answers.

- Question 7 gave the beginning base-10 block display for a subtraction of two decimal 100ths and asked students to explain how to complete the process using the blocks. It was disappointing that 82.6 percent of the students in the provincial sample were not even able to begin to explain the trading and removing of blocks necessary to complete the subtraction.

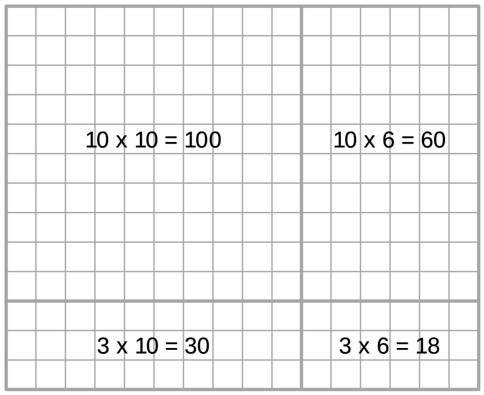
Additional Suggestions

The following suggestions, based on the results of the 2001 Elementary Mathematics Assessment, are provided for teachers to assist their students to improve their mathematical skills and understanding.

1. The result of Part 7: Question 7 and the result of partitioning the array picture for the product of 15 and 17 (Part 3: Question 3(B)) clearly point to the necessity for more modeling of procedures with concrete materials. Such modeling brings clarity to the connection between what students do with the symbols and what they do with the concrete materials and pictures. Modeling should not be just a demonstration or a one-time thing—it is an expectation that all students use concrete materials to develop procedures before they learn the symbolic recording of these procedures and that they continue to connect these two representations.

   Addition and subtraction should include modeling of whole numbers up to 1000s and of decimals to 1000ths using the base-10 blocks. This modeling can be done in a variety of ways such as in groups with students taking turns, in partners with one partner modeling with the blocks while the other partner records the symbols, and individually with the student explaining in writing what was done with the blocks.

Multiplication should be modeled as arrays up to and including two two-digit numbers using base-10 blocks and base-10 grid paper (see appendix of *Report on Field Tests: What We Learned*). An example on ordinary grid paper is shown below for 13 x 16.



$$\begin{array}{r} 16 \\ \underline{\times\, 13} \\ 18 \\ 30 \\ 60 \\ \underline{100} \\ 208 \end{array}$$

2. The approach to solving and creating story problems for each of the four operations should be front and centre in the study of these operations. This approach is in stark contrast to the treatment of these operations in former text resources, where a very few story problems followed 25–35 exercises designed to practise a specific skill. Since most story problems use the procedure just rehearsed, many students merely took the numbers they saw in the problem and performed that operation without reading the problem.

   By contrast, story problems should be used to introduce students to an operation concept. Students should solve story problems by modeling with concrete materials and discussing the processes they used. Next, students should record their solutions to story problems by drawing pictures and/or writing descriptions of the process used in their modeling. They should then create simple story problems similar to the ones they have been solving. Finally, the symbols should be connected to the story problems, the pictures, and the concrete material manipulations—emphasizing that symbols are a form of communication.

   As the study of the concept progresses, students should be exposed to a wider variety of related story problems and to story problems of a more complex nature, with ongoing opportunities to create their own story problems. Students should develop an awareness of the patterns of structure inherent in related story problems. As well, students should have opportunities to compare and contrast current story problems with those previously studied in order to detect the often subtle differences in language—a practice that reinforces the necessity for careful and deliberate reading of story problems.

3. Teachers must plan for their students a careful and deliberate transition from concrete/pictorial modeling of procedures to working exclusively with symbols. This might be referred to as the *visualization* stage where students manipulate the symbols while making pictures of a model in their minds and talking about that model.

   - For example, while finding the product of 12 and 19, students could visualize the array of 12 rows with 19 in each row and the partitioning of this array into the four arrays—10 x 10, 10 x 9, 2 x 10, and 2 x 9. As students multiply in each step, they would refer to the piece of the array that this sub-product is finding and to the sum of these four arrays for the total of the array. Teachers should make reference to the associated model when doing procedures on the board or overhead projector.

   The number of subtraction errors that students made in the assessment suggests that they are not visualizing the model of the process as they perform each step.

   - For example, to find 3.7 – 0.812, the three most common errors made by students are (1) to put 0.812 above 3.7, believing it to be larger, (2) to right-justify the question so that the 2 and 7 are aligned rather than the decimal points, and (3) to subtract the smaller digit from the larger in each place value.

   One, two, or all of these types of errors were observed in students' responses to questions in the assessment. To model the above example with base-10 blocks, students would make a display of three large cubes and seven flats to represent the 3.7 (minuend), and because it is a subtraction question, they would know that eight flats, one rod, and two small cubes—the 0.812—would have to be removed from this display. This modeling of the process would reinforce (1) what has to be traded to have the types of blocks needed to be removed, (2) what is actually being subtracted, and (3) what place values should be aligned. At the *visualization* stage, students might initially draw pictures of the blocks to represent the minuend and refer to the trading and removing of these blocks as they perform the algorithm. When the students are comfortable, they might just visualize and talk about the blocks as they perform the procedure.

   Having students write answers to questions such as, "Explain how you would model 0.35 – 0.249 using base-10 blocks," should be part of the development and the assessment of procedures. A greater degree of awareness of the process will help to curtail the types of errors mentioned and aid in the long-term retention of algorithms.

4. With the central focus of the curriculum placed on problem solving, there are definite implications for the teaching of reading strategies in mathematics. One such strategy is referred to as the *Three-Read Strategy*. As suggested by its name, students are encouraged to read a story problem three times to ensure that they fully understand the problem before they make attempts to solve it. Each read of the problem has a specific purpose—(1) the first read is to get a general impression of what the problem is about without trying to pick up the details, (2) the second read is to gather and make mental images of the details focussing on what is given and what question is being asked, and (3) the third read is to check that these details have been correctly noted. After each read, students should stop and have internal conversations about the problem.

   In order to teach the *Three-Read Strategy*, teachers should exaggerate each step as they model it. When they have students practise the strategy, teachers should ask questions that simulate the kinds of questions that students should be asking themselves in their internal conversations. In every classroom, an ongoing discussion of this *Three-Read Strategy* must be conducted and many students will need to be reminded to use the strategy.

   - For example, suppose the following story problem were put on the overhead projector to use to teach the *Three-Read Strategy*:

     The Roper children went to the fair ground at the exhibition. They had $20 to spend on rides. The four children went on five rides. Each of the rides costs $0.75 per person. How much money will the children have left over?

   The teacher should read the problem out loud as students follow along on the image. Then, with the overhead projector turned off, the teacher asks one student (and invites others to follow) to explain what the problem is about without mentioning any numbers. Next, the teacher turns on the overhead projector and reads the problem again, stopping after each statement that gives information.

   - For example, after reading the second sentence, the teacher would say, “This is one thing we know—they have $20 to spend.” After reading the third sentence, the teacher says, “The second thing we know is that there are four children, and the third thing we know is that they went on five rides.” After reading the next sentence, the teacher says, “The fourth thing we know is that each ride costs 75 cents.” Finally, after reading the last sentence, the teacher says, “What we need to find out is how much money they will have left over after these rides.”

- The teacher then turns off the overhead projector and asks the class questions, calling upon specific students to answer. Examples of these questions are: How many things were we given in this story problem? How much money did they have to spend? How many children were there in the family? How many rides did they go on? What did the rides cost? What are we supposed to find out?

- Finally, the teacher turns the overhead projector back on and reads the problem once more for students to check the four givens and the question. The teacher proceeds, confirming each of the four givens and the question and then explains that they should now understand the problem and should think about possible strategies to solve it.

This *Three-Read Strategy* would have to be modeled several times and in a variety of ways. If, in their ongoing problem-solving, students say that they don't understand a problem, the teacher should check that they have used this strategy by asking questions such as, How many things were you given in this problem? What are they? What are you supposed to find out? If students cannot answer these questions, the teacher should not even begin to help them with a solution strategy; rather, the students will need to re-read to get the necessary information.

5. A significant part of learning to solve problems is to learn about the problem-solving process. It is generally accepted that the problem-solving process consists of four steps—understanding the problem, devising a plan, carrying out the plan, and looking back. Beginning in grade primary, students should experience this process through direct and overt modeling of day-to-day problems. From about grade 3 onwards, teachers should plan some lessons that deal specifically with aspects of the problem-solving process.

   - For example, the *Three-Read Strategy* discussed previously in (4) pertains to the first step of the process—understanding the problem. Not understanding the problem fully before trying to solve it results in many errors and is a source of frustration for students when they realize that they missed an important detail and have to redo much of their work.

   Another important step of the problem-solving process is devising a plan for solving the problem. Students should have had experiences with many strategies in the ongoing development of concepts, such as concrete modeling, drawing pictures, performing operations, and looking for patterns. Other strategies that should be discussed by students and modeled include: guess-and-check, making a table or chart, making an organized list, working backwards, trying a simpler problem, using logical reasoning, changing point of view, and writing an open sentence.

To teach any one of the above strategies, teachers should choose a few problems for which the strategy very obviously works; assign one of these problems for the students to solve, reminding them to read it three times; have students share their solutions, hoping that at least one of the students suggests the strategy to be highlighted; discuss this strategy in detail with the students; and assign the other problems for students to solve, suggesting that they use the highlighted strategy if they can.

Note: There are many ways to solve a problem, but in a lesson designed to highlight a specific strategy, teachers should encourage all students to try to use this strategy until they understand it. The students can add each new strategy to their growing repertoire of strategies. Afterward, if similar problems are assigned, students should feel free to use whatever strategy they wish. A useful suggestion is to add the name of the particular strategy being highlighted to a displayed class “Strategy List.”

The development of such a strategy list could be a focus in the second step of the problem-solving process. Once students understand the problem, they need to think about strategies they could use to work with the given information in order to answer the question(s) asked in the problem. If students have trouble at this step, the teacher can direct their attention to the strategy list and encourage them to think of a possible approach using a strategy from this list.

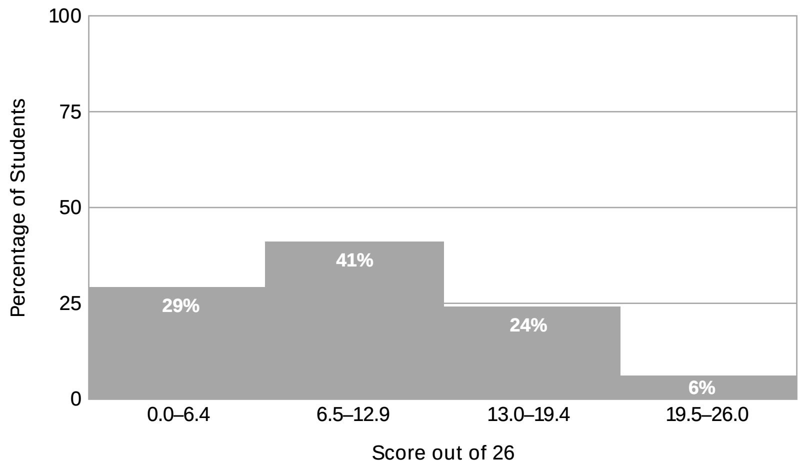
Part of the modeling of the second part of the problem-solving process should include the asking of questions that are typical of the types of questions students should be asking themselves. For example, consider the story problem used in (4) about the Roper children going on rides at the fair ground. Students could be asked questions such as,

- What general strategy do you think we could use given the information we have?
- Why do you think we were told there were four children?
- Why were we told that they took five rides?
- What makes sense for us to do with the 75 cents per ride?
- What are we going to do with the $20? Why?
- Is there anything we were given that we don’t really need to know?

Provincial Sample Quartile Results for Category 3

The histogram below presents the percentage of students in the provincial sample that scored in each quartile of the 26 points accounted for by the questions dealing with operation concepts and applications.

Operation Concepts and Applications



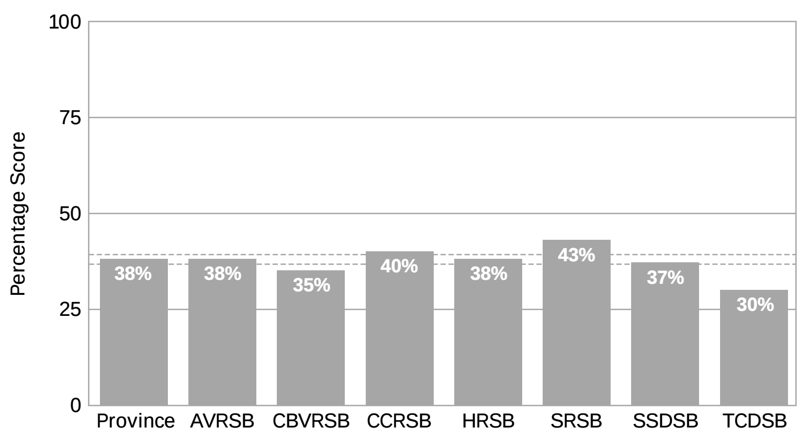
Category 4
Measurement Concepts and Applications—GCO D

The questions in the category Measurement Concepts and Applications accounted for 15 points out of the 125 total points on the assessment. The distribution of the questions across the assessment and the results are shown in the table on the next page.

The questions in this category assessed students' abilities at determining area, perimeter, and volume; at applying area and perimeter; and at estimating length, capacity, and angle measure, using appropriate units.

The graph below shows the provincial mean percentage score for measurement concepts and applications as 37.5 percent and the school board mean percentage scores ranging from a low of 29.9 percent to a high of 43.3 percent.

Measurement Concepts and Applications



Percentage scores rounded

Table 4: Measurement Concepts and Applications

Part	Question Number	Point Value	Target SCO	Description of Skill/Knowledge/Concept	Mark: %Achieving*
2	12	1	4D4	Find the volume of a rectangular prism	1: 35.2%
	13	1	4D2	Recognize perimeter as a form of measure	1: 65.2%
	14	1	3D4	Find the area of the largest triangle on a grid	1: 50.5%
	15	1	5D4	Recognize the equivalence of 500 mL and 0.5 L	1: 28.1%
4	51	0.5	3D1	Recognize an estimate of length (dm)	1: 26.7%
	52	0.5	3D2	Recognize the capacity of a teaspoon (mL)	1: 51.4%
	53	0.5	3D2	Recognize the capacity of a soup can (mL)	1: 37.6%
	54	0.5	5D7	Recognize an estimate of angle measure	1: 35.6%
6	2	3	3D8 4D9	Create a shape with three conditions including an area of 12 square units	3: 14.7% 2.5: 5.9% 2: 3.9% 0: 36.5%
	6	3	5D2	Determine the estimate of an irregular shape on a grid	3: 15.5% 2.5: 18.1% 2: 10.0% 0: 24.0%
7	10	3	5D1 5D5	Draw rectangles with perimeter and area restrictions	3: 2.8% 2.5: 1.6% 2: 18.7% 0: 56.6%

*These percentages are based on the provincial sample of grade 5 students.

Comments

It is interesting to note that in the 2001 Elementary Mathematics Assessment, more students were able to determine the area of an irregular shape on a grid (Part 6: Question 6)—43.6 percent—and to recognize the triangle of largest area possible on a grid (Part 2: Question 14)—50.5 percent—than were able to draw a shape with a specific area on a grid (Part 6: Question 2)—24.5 percent—or to draw rectangles with an area restriction (Part 7: Question 10)—23.1 percent.

Previous Suggestions

There are many comments and suggestions on pages 13–15 of *Report on Field Tests: What We Learned* for the 2000 mathematics field test that continue to be appropriate for the consideration of teachers following the 2001 Elementary Mathematics Assessment. Teachers should consult the earlier report and the pages specified for information related to (1) the components of teaching any measurement concept, including the establishment of referents, (2) clarifying the common confusion between area and perimeter, and (3) teaching angle measurement.

Additional Suggestions

The following suggestions, based on the results of the 2001 Elementary Mathematics Assessment, are provided for teachers to assist their students to improve their mathematical skills and understanding.

1. Teachers should not expect that students will memorize conversions among all the metric units; rather, they should expect students to remember some of the most frequently used conversions through their experiences in measuring and through familiarity with their referents for these units. Examples are given below.

   - Students should be provided with learning experiences with a metre stick so that the 100-cm and 1000-mm equivalents are almost automatic and easily visualized.

   - Students should view a rod in the base-10 blocks as a “decimetre stick”; thus they easily see the 10-cm equivalence. Similarly, by placing 10 of these rods alongside a metre stick, students can establish the equivalence of 1 dm and 0.1 m.

   - In order for students to appreciate the size of 1 kilometre, they should be provided with experiences that expand upon metre measurements, establishing lengths of 1000 of these metres. For example, if the school driveway is 100 m long, then 1 km would be 10 of these driveways.

     Every opportunity to mention these equivalencies in linear measurement should be taken. For example, if students measure an object and get 3 dm, they should be asked, How many centimetres would this be? What part of a metre?

   - For units of capacity, students should know the 1-litre and 1000-millilitre equivalency with the relationship between the small and large cubes in the base-10 blocks as the visual anchor. Certainly, since the large cube is 1 litre and the small cube is 1 millilitre, it would seem to be the most obvious referent for work with these units. Since these units are commonly used in society in the sales of soft drinks, milk, gasoline, and other liquids, there is another strong support for learning units of capacity.

While we might refer to a quantity being a half litre, students should understand that it must be written as a decimal—as are all metric units—so, they could read 0.5 L as “one-half litre” or as “five-10ths litre.” Experiences representing litres and parts of a litre using the various base-10 blocks and discussing what is represented in both litres and millilitres will help students visualize the relationships.

- Similarly, the units for volume—cubic centimetre (cm^3) and cubic decimetre (dm^3)—are easily visualized and connected to units of capacity through the same base-10 blocks.

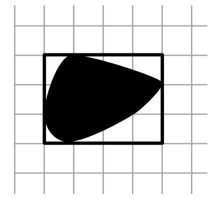
- For units of area, students should get to know that 100 cm^2 is equivalent to 1 dm^2 through the area of the top face of a flat in the base-10 blocks.

- For units of mass, students should know that 1000 grams is equivalent to 1 kilogram. The commercial plastic centicubes have been designed so that each cube has a mass of 1 gram; therefore, these cubes are helpful in the development of understanding of units of mass. The development of estimation of mass requires the use of the sense of touch rather than of sight; therefore, students need many hands-on experiences lifting objects and comparing them to known masses.

2. Geoboards, square dot paper, and grid paper should be used in the teaching of perimeter of shapes, because they provide opportunities to concentrate on what is actually being measured and, at the same time, provide opportunities for discussion of the areas of these shapes. Teachers should be watchful for the common errors students make—counting the pegs around geoboard shapes rather than the distance between these pegs, and counting the squares around the inside of shapes on grid paper rather than the outer sides of these squares. When perimeter questions are demonstrated, they should be modeled so it is very clear what is being measured.

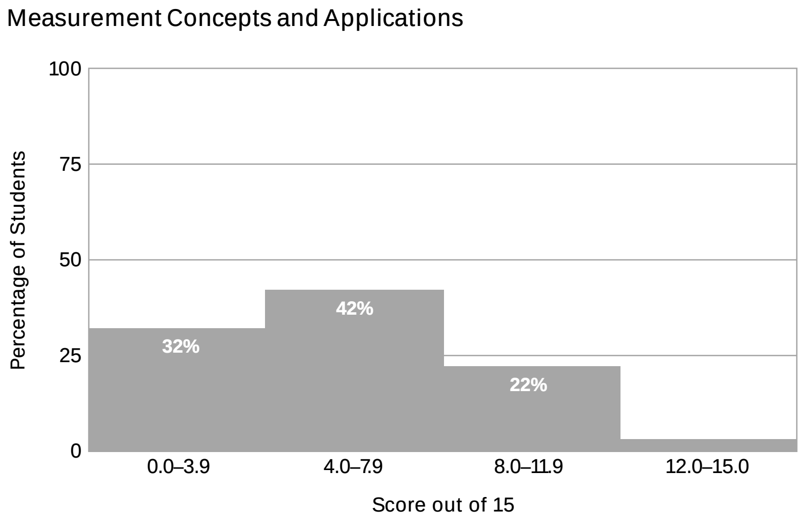
3. When students learn to determine the area of irregular shapes, they must be taught to realize that they are estimating when they combine two or more partial squares to get whole number square units. Students should not be referring to partial squares as “half” unless they are half. Students should experience two strategies for determining the areas of irregular shapes—counting full squares and combining partial squares in the interior of the shapes, and drawing a rectangle to enclose the shape (see diagram below) and subtracting full and combined partial squares between the rectangle and the irregular shape.

Area of rectangle = 3 x 4 = 12 sq. units
Area of Irregular Shape ≈ 12-4.5 ≈ 7.5 sq units



Provincial Sample Quartile Results for Category 4

The histogram below shows the percentage of students in the provincial sample that scored in each of the quartiles of the 15 points accounted for by the questions dealing with measurement concepts and patterns.



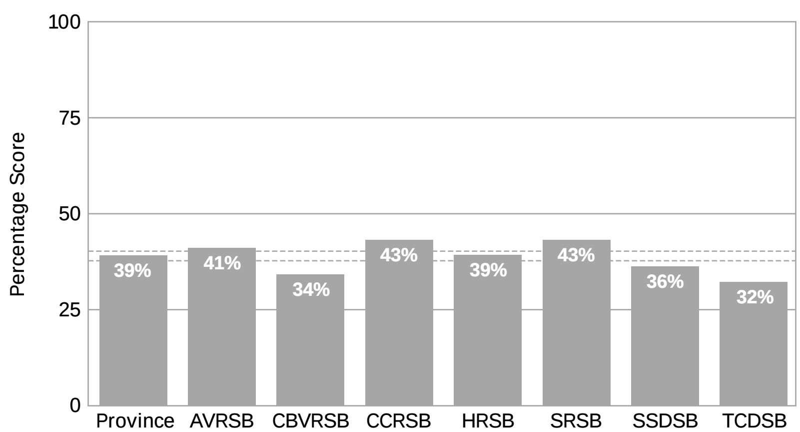
Category 5
Geometry Concepts and Spatial Sense—GCO E

The questions in the 2001 elementary mathematics assessment for the category Geometry Concepts and Spatial Sense accounted for 26 out of the 125 total points on the assessment. The distribution of the questions across the assessment and the results are shown in the table on the next page.

The questions in this category assessed students' understandings and abilities in two-dimensional, three-dimensional, and transformational geometries as well as some aspects of spatial sense.

The graph below shows the provincial mean percentage score in this category as 38.7 percent and the school board mean percentage scores ranging from a low of 32.3 percent to a high of 43 percent.

Geometry Concepts and Spatial Sense



Percentage scores rounded

Table 5: Geometry Concepts and Spatial Sense

Part	Question Number	Point Value	Target SCO	Description of Skill/Knowledge/Concept	Mark: % Achieving*
2	11	1	4E3	Interpret an isometric drawing	1: 71.2%
	21	1	5E1	Recognize a net of a triangular prism	1: 62.7%
	22	1	4E10	Visualize the faces of three-D shapes	1: 44.5%
	23	1	3E9	Recognize lines of reflective symmetry	1: 45.6%
	24	1	4E11	Recognize described translation and reflection	1: 35.3%
	25	1	4E11	Recognize the result of described rotation	1: 43.5%
	26	1	4E11	Recognize the result of the translation, given arrows	1: 31.7%
	27	1	3E3	Recognize a set of congruent shapes	1: 49.8%
	28	1	4E8	Recognize a shape, given relationships	1: 42.4%
	29	1	5E8	Recognize an incorrect property of squares	1: 41.8%
	30	1	4E5	Recognize a shape built from triangles	1: 39.0%
5	2	1	4E11	Draw the result of a described reflection on grid paper	1: 73.9% 0.5: 5.7%
	3	1	3E10	Draw the result of a described rotation on grid paper	1: 6.9% 0.5: 11.0%
	4	1	5E13	Dissect a shape into three described shapes	1: 63.4% 0.5: 14.0%
6	4	3	5E3	Create a three-D shape with cubes that meet criteria and draw an isometric image of the result	3: 20.6% 2.5: 6.9% 2: 10.5% 0: 38.3%
7	9	3	4E8 4E12 5E8 5E11	Describe six ways that a given rectangle and square (on grid paper) are alike	3: 1.0% 2.5: 4.2% 2: 11.7% 1: 18.3%
8	14	6	3E4 3E9	Create three named shapes with pattern blocks, creating a shape that meets three stated criteria	6: 2.2% 5: 3.4% 4: 4.3% 0-1: 68.1%

*These percentages are based on the provincial sample of grade 5 students.

Comments

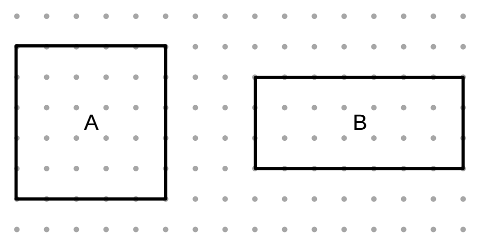
In transformational geometry, students were more able to recognize and represent reflections and to recognize a described rotation (Part 2: Questions 23, 24, 25, and Part 5: Question 2) than they were able to draw a rotation (Part 5: Question 3). It appears from the assessment results that many students have not had experiences drawing described rotations.

Clearly, students were more able to work with an isometric drawing of a three-dimensional shape than they were able to produce such a drawing (Part 2: Question 11 and Part 6: Question 4). It is interesting to note, however, that some improvement has been made since the field-testing of isometric drawings in 2000.

As was observed in the field-testing of assessment questions, students had difficulties making specific polygons with pattern blocks and solving a pattern block problem (Part 8: Question 14). The results of Part 8: Question 14 and Part 2: Question 30 suggest that students are not working enough with pattern blocks. So many geometry concepts, spatial abilities, and other mathematics concepts can be developed using pattern blocks. Pattern blocks should be one of the primary concrete materials used at all grade levels. The aforementioned pattern block questions included a problem that required students to meet three conditions. The meeting of multiple conditions appeared to be difficult for students in the pattern block questions as well as in other questions on the assessment.

From Part 7: Question 9, it is evident that students are not bringing into their comparisons of shapes many of the properties that they would have studied. The rectangle and square in Question 9 are alike in at least nine ways:

1. They are both quadrilaterals.
2. They both have two pairs of equal opposite sides.
3. They both have their opposite sides parallel.
4. They both have four right angles.
5. They both have diagonals of equal length.
6. The diagonals in both shapes bisect each other.
7. They both have reflective symmetry.
8. They both have rotational symmetry.
9. They both have a perimeter of 20 units.



Question 9 required students to address six ways the shapes were alike; however, only 5.2 percent of the students sampled were able to provide five or six of these ways.

Previous Suggestions

The *Report on Field Tests: What We Learned* pp. 15–19 provided important comments and suggestions about (1) concave, convex, and regular polygons; (2) relationships between lines and line segments; (3) using known properties of shapes in making comparisons; (4) recognizing and visualizing transformations; (5) combining and dissecting shapes; and (6) interpreting and creating isometric drawings. Teachers are encouraged to review this portion of the report.

Additional Suggestions

The following suggestions, based on the results of the 2001 Elementary Mathematics Assessment, are provided for teachers to assist their students to improve their mathematical skills and understanding.

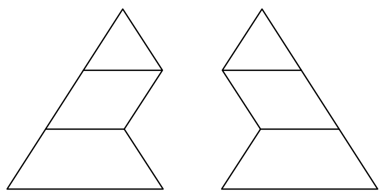
1. Of the plane transformations, rotations are the most difficult to recognize, describe, and represent. Each rotation has a centre of rotation, an angle of rotation, and a direction of rotation, all of which need to be considered. Students should learn to represent rotations of two-dimensional shapes in grade 3 with centres at the vertices of the shapes and angles of one-quarter turn, one-half turn, and three-quarters turn, in both clockwise and counter-clockwise directions. These rotations are best done on grid or square dot papers with tracing paper used to assist in the drawings. (See Grade 3, SCO, E10 in the Atlantic Canada mathematics curriculum.) In grade 4, rotations are extended by considering centres along straight line extensions of the sides of the shapes.

2. Students should solve a variety of problems using pattern blocks and discuss the results using as much related mathematics vocabulary as possible.

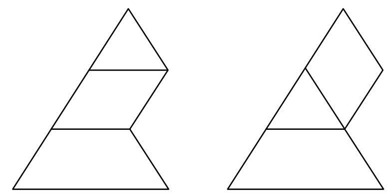
   Each of these blocks should be referred to by the name that is appropriate for the grade level. For example, in grade 2 the blocks are referred to as triangle, square, trapezoid, hexagon, blue rhombus, and tan rhombus; however, by grade 5, the triangle, trapezoid, and hexagon should be called equilateral triangle, isosceles trapezoid, and regular hexagon. As well, the polygons that result from combining these blocks should also be named correctly.

An example of a problem-solving small-group activity that also highlights spatial sense is described below.

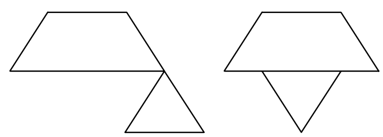
Ask students working in groups of three or four to make as many different polygons as they can using three pattern blocks—the isosceles trapezoid, the blue rhombus, and the equilateral triangle. Place each polygon in the middle of the work space so everyone can see it. Before groups begin, explain what is meant by different; that is, a polygon is not different from others if it is a rotated or reflected image of one of them **and** if it is the same shape made by a different configuration of the blocks.



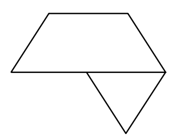
These are the same pentagon because they are reflected images of one another.



These are the same pentagon made by arranging blocks in a different way.



These two are not permitted.



This one is permitted.

Also explain that when blocks are connected it must be along their sides and must share at least one vertex.

When groups are satisfied that they have all the different polygons, they should trace around the outsides of the shapes to record them on paper. These polygons could be cut out and compared to one another to make sure they are all different. (**Answer**: There are 12 different polygons.)

Activities such as the one above can serve as a springboard to other activities with other purposes. For example, this activity will generate 12 different polygons that can be used to reinforce a number of other concepts when

- students name the polygons
- students sort them into sets of convex, concave, and regular polygons
- students identify the polygons that have reflective and/or rotational symmetry
- students visualize where the three blocks would be placed inside each polygon
- students find how many ways each polygon could be made by different configurations of the blocks
- students find which concave polygons could be made convex by adding one more pattern block
- students find ways to make the non-symmetrical polygons symmetrical by adding the fewest number of pattern blocks
- students make larger polygons by combining two or more of these 12 polygons
- students check to see which of the 12 polygons can be made with two trapezoids or with three blue rhombuses

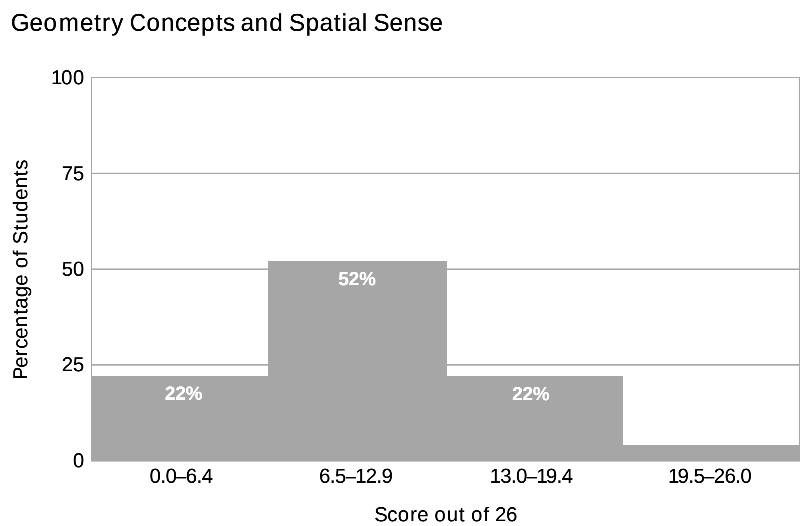
3. In geometry, as well as in other areas, students should be asked to solve problems that require them to satisfy two or more conditions. Two examples are described below.

   - In grades 3 or 4, students could be asked to use three pattern blocks to create a nonagon (9-gon) with one line of reflective symmetry. (Many students will begin with three pattern blocks but will forget this condition as they add additional blocks to make a nonagon or to get one line of symmetry.) As in all problem solving, students should be reminded to look back when they are finished to make sure they have answered the question that was asked.

   - In grades 5 or 6 students could be asked to draw on grid paper a quadrilateral with two lines of reflective symmetry and with an area of 24 square units.

Provincial Sample Quartile Results for Category 5

The histogram below shows the percentage of students in the provincial sample that scored in each of the quartiles of the 26 points accounted for by the questions dealing with geometry concepts and spatial sense.



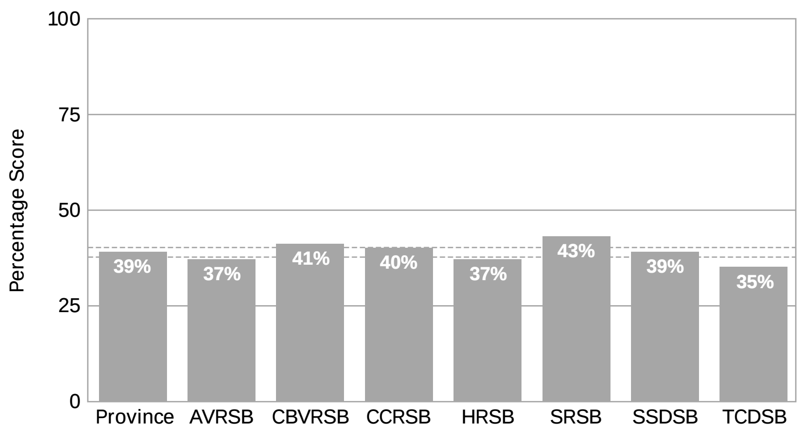
Category 6
Data Management and Probability—GCO F and GCO G

The questions in the category Data Management and Probability accounted for 22 points out of the 125 total points on the assessment. The distribution of the questions across the assessment in this category and the results are shown the table on the next page.

These questions assessed students' abilities at constructing and interpreting different types of graphs—tally chart, pictograph, line graph, scaled bar graph, and stem-and-leaf plots; at working with means of data; and at working with probabilities in coin, spinner, and die situations.

The graph below shows the provincial mean percentage score for this category of the assessment as 38.7 percent and the school board mean percentage scores ranging from a low of 35.3 percent to a high of 43.1 percent.

Data Management and Probability



Percentage scores rounded

Table 6: Data Management and Probability

Part	Question Number	Point Value	Target SCO	Description of Skill/Knowledge/Concept	Mark: % Achieving*
2	8	1	5G2	Recognize probability notation for a coin toss	1: 51.4%
	9	1	4G3	Recognize the most likely result for a spinner	1: 85.3%
	10	1	5G1	Recognize the most likely set of results for a die	1: 37.9%
	16	1	4F3	Interpret a tally chart	1: 72.8%
	17	1	4F3	Interpret a pictograph	1: 49.0%
	18	1	4F4	Know the coordinates of a point	1: 51.7%
	19	1	5F4	Recognize a line graph for a stated relationship	1: 40.5%
	20	1	4F3	Interpret a stem-and-leaf plot	1: 32.3%
5	9	1	3G1	Determine the expected result of a given spinner	1: 19.0% 0.5: 10.5%
	10	1	4F3	Solve a problem using information on a given scaled bar graph	1: 55.9% 0.5: 4.3%
8	12	6	5F5 5F6 4F3	Create a stem-and-leaf plot given data; recognize the effect of new data on a mean; make two observations from the plot	6: 1.3% 5: 6.5% 4: 20.2% 0-1: 52.8%
	13	6	3F3 4F3 4F7	Create a scaled bar graph from given data; find a fractional answer; work with a mean	6: 1.0% 5: 5.8% 4: 12.8% 0-1: 42.2%

*These percentages are based on the provincial sample of grade 5 students.

Comments

Examining the results of the selected-response questions in Part 2 of the assessment provides evidence that students are more able to interpret a tally chart, a pictograph, and a line graph than they are a stem-and-leaf plot.

Given the student response to Part 8: Question 12, we can assume that the stem-and-leaf plot is not getting the treatment expected in grades 4 and 5 classes that is required by the Atlantic Canada mathematics curriculum. In fact, some students drew pictures of stems and leaves, while others wrote that they had not been taught this type of graph. The stem-and-leaf plot is not a difficult graph, and it is popular because it is less abstract than other types of graphs. The reason for the disappointing results may also be the fact that this is a new type of graph for many teachers and is not well represented in some of the listed resources at grades 4 and 5.

While the students' attempts at drawing a scaled bar graph (Part 8: Question 13) were somewhat better than in the field tests, the students continued to disregard the central premise that a graph has to tell the whole story by making some, or all, of the following errors:

- omitting the title
- not labelling the axes
- not labelling the bars
- not clearly indicating the scale on the lines
- not consistently using the scale
- not labelling zero
- not leaving spaces between the bars

Since a bar graph represents quantities of discrete data (such as types of books, kinds of ice cream, and kinds of pets), there should be spaces between the bars. There is another type of graph called a histogram, used to represent quantities of continuous data (such as measurements of height, mass, and time where the horizontal axes would indicate intervals of these measurements); thus, there should be no spaces between the bars in this type of graph.

While the 2001 Elementary Mathematics Assessment did not put a lot of emphasis on line graphs, it is interesting to note that only 40.5 percent of the students were able to recognize the line graph associated with a stated relationship (Part 2: Question 19) and only 51.7 percent of the students could correctly identify a point on the coordinate system when given its coordinates (Part 2: Question 18). The knowledge required for the latter question would be basic in working with line graphs and raises the concern about how widespread is the neglect in teaching this type of graph.

It is also interesting to note that 85.3 percent of students were able to discern that the largest region on a spinner would be the most likely result (Part 2: Question 9); however, less than 30 percent of them were able to give a correct answer when asked to predict the number of times blue occurs in 24 spins of a spinner on which the colour blue appeared on two out of four equal sectors (Part 5: Question 9).

Previous Suggestions

Teachers should consult pages 19–22 of *Report on Field Tests: What We Learned* for comments and suggestions about

- the construction of stem-and-leaf plots
- the difference between reading graphs and making observations about them
- the reading, interpreting, and construction of line graphs
- the effects of changes of data on its mean
- the misconceptions regarding probability applied to spinners

Additional Suggestions

The following suggestions, based on the results of the 2001 Elementary Mathematics Assessment, are provided for teachers to assist their students to improve their mathematical skills and understanding.

1. When students are introduced to a new type of graph, it should be through a context or activity for which this graph would be an ideal way to organize or display the resultant data.

   - For example, if students are going to be introduced to the line graph, it might be after they have monitored the growth of bean plants over three weeks and kept the daily results in a table. Discussing how this collected data can be displayed as a “picture” should result in the development of a line graph where the horizontal axis would show days (either capital letters for the days of the week or numbers 1, 2, 3, 4, ... for the days of monitoring) and the vertical axis would show the growth (0, 1, 2, 3, 4, ... for measurements in centimetres).

2. Students should make observations about their own graphs. In the example above, when their line graphs are plotted, the students should make observations about the growth of their plants such as the periods of greatest, least, and no growths, and how long it took to reach half the maximum growth recorded.

3. Graphs provide opportunities to integrate other mathematics concepts in other strands, such as number and measurement, and concepts in other disciplines, such as social studies and science. The graphs can also be reinforced in these other strands in mathematics and in these other disciplines. Many newspapers use a variety of graphs in their articles and presentations—these can be sources of graphs for discussion and to show how graphs are used in the world around us. (*USA Today* is a particularly rich source of graphs.)

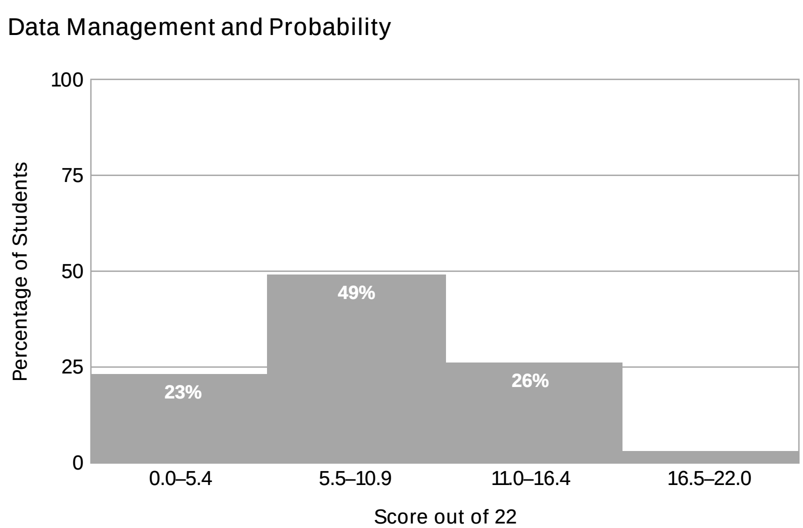
4. Teachers must emphasize to students the importance of including a title and labels on all graphs. A graph is a form of communication—a picture that must tell the complete story on its own without reference to a written explanation, a table of values, or any other device. Activities to emphasize this convention may involve displaying graphs that have missing labels and asking students to do one or a combination of the following:

   - answer given questions, at least some of which are impossible with a label missing
   - write an explanation of the graph, which will underscore the communication problem of missing labels
   - critique the graph, suggesting improvements that need to be made
   - provide suggestions for missing labels so the graph tells a complete story

5. After a particular graph has been taught, students should be expected to demonstrate what they have learned by creating a graph on their own without assistance from others. By creating their own graphs, students can learn from their mistakes.

Provincial Sample Quartile Results for Category 6

The histogram below shows the percentage of students in the provincial sample that scored in each of the four indicated intervals of the 22 points accounted for by the questions dealing with data management and probability.



Conclusion

Overall, the results of this first program assessment for the Atlantic Canada mathematics curriculum in the elementary grades are disappointing. The results indicate that there is more to be done in the process of implementing the prescribed curriculum on a whole-school basis.

There may be other factors, besides curriculum implementation, that have influenced the results. For example, there was no stake for the students who wrote the assessment and thus their commitment to do their best may have been minimal. As well, this was the first assessment of its kind, and Nova Scotia students have not had experience with a comprehensive provincial assessment. It is difficult to report with certainty on the extent of all the factors that may have influenced the results; nevertheless, it can be stated that most students did attempt the questions in this assessment of the mathematics program and therefore any inferences that are made about the implementation of the curriculum are based on the quality, or lack thereof, in student responses.

This response to the 2001 Elementary Mathematics Assessment, by way of general comments and specific suggestions, recommends steps that must be taken for students to receive a coherent mathematics program, and it pinpoints many specific problems—with suggestions for teaching and learning—within the six categories based on the general curriculum outcomes. While the results of this assessment will serve as baseline data against which results of future assessments will be compared, significant improvement in student achievement will occur only if all stakeholders take action using the information contained within this study document.

In the process of promoting the improvement of mathematics education in Nova Scotia, teachers have a key role. Individual teachers at every grade level must take responsibility for their part in the continuum of mathematics education, as outlined in the Atlantic Canada mathematics curriculum. Teachers must see their place in the ongoing development of the network of mathematical ideas in the minds of their students. Teachers cannot do this alone. They need the support of others. The school principal has a critical supporting role to facilitate a school plan for a whole-school approach to the teaching and learning of mathematics. The school board must likewise encourage the efforts of each school in this regard.

There are many excellent resources available to assist teachers to effectively teach mathematics skills and concepts with the goal of helping their students learn the mathematical ideas contained in the prescribed elementary mathematics curriculum. The challenge for all is to maximize the use of the available resources in planning, organizing, and delivering a coherent mathematics program that is focussed on the goals and outcomes of the Atlantic Canada mathematics curriculum.

Notes: